UNIFIED COUNCIL
Foundation for Success

UNIFIED COUNCIL OLYMPIADS

nstse
Test • Assess • Achieve

NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 11 (PCM)
Question Paper Code : UN489

KEY

1. A	2. A	3. B	4. C	5. B	6. A	7. B	8. A	9. D	10. B
11. C	12. B	13. C	14. B	15. B	16. A	17. C	18. A	19. D	20. B
21. C	22. B	23. D	24. C	25. C	26. C	27. Del	28. D	29. D	30. B
31. C	32. D	33. C	34. C	35. A	36. C	37. D	38. C	39. A	40. A
41. A	42. D	43. C	44. D	45. C	46. B	47. A	48. D	49. A	50. B
51. B	52. A	53. A	54. C	55. C	56. A	57. C	58. C	59. B	60. B

SOLUTIONS

MATHEMATICS

01. (A) If A = {1, 2, 3}, B = {2, 3, 4}, C = {1, 2, 3, 5},

then $A \cap B = \{2, 3\} \subseteq C$,

$A - C = \varnothing \subseteq B$

But $A \not\subseteq B$

$\therefore$ Option (A) is not true

$(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C$

$\therefore$ Option (B) is true

If $A - B = \{1\} \subseteq C$

$\therefore (A - B) \subseteq C$ and $(A \cap B) \subseteq C$

$\therefore (A - B) \cup (A \cap B) \subseteq C$

i.e., $A \subseteq C$

$\therefore$ Option (C) is true

$\varnothing \neq A \cap B \subseteq C, A \cap B \subseteq B$

$\Rightarrow A \cap B \subseteq B \cap C$

$\Rightarrow B \cap C \neq \varnothing$

Hence, option (D) is true

02. (A) Multiple of 4 are 4, 8, 12, 16, 20; Multiples of 3 are 3, 6, 9, 12, 15, 18

4, 8, 12, 16, 20 mapped to 3, 6, 9, 12, 15, 18 in $6 \times 5 \times 4 \times 3 \times 2 = 6!$ ways

Remaining 15 numbers mapped in 15! ways

$\therefore$ The number of onto (bijective) functions is 6! × 15!

03. (B) $|z-3|+|z+3|=8$

$\Rightarrow |(x-3)+iy|+|(x+3)+iy|=8$

$\Rightarrow \sqrt{(x-3)^2+y^2}=8-\sqrt{(x+3)^2+y^2}$

$\Rightarrow (x-3)^2 + y^2 = 64 + (x+3)^2 + y^2 - 16\sqrt{(x+3)^2+y^2}$

$\Rightarrow -12x - 64 = -16\sqrt{x^2+y^2+6x+9}$

$\Rightarrow 3x + 16 = 4\sqrt{x^2+y^2+6x+9}$

$\Rightarrow 9x^2 + 96x + 256 = 16(x^2 + y^2 + 6x + 9)$

$\Rightarrow 7x^2 + 16y^2 = 112$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{7}=1$ which is an ellipse such that a = 4, b = $\sqrt{7}$

Let $z_1 = x_1 + iy_1$ Now arg

$z_1 = \frac{\pi}{6} \Rightarrow \text{Tan}^{-1}\frac{y_1}{x_1}=\frac{\pi}{6}$

$\Rightarrow \frac{y_1}{x_1}=\frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}y_1 = x_1$

z_1 lies on the ellipse

$\Rightarrow \frac{x_1^2}{16}+\frac{y_1^2}{7}=1 \Rightarrow \frac{3y_1^2}{16}+\frac{y_1^2}{7}=1$

$\Rightarrow 37y_1^2 = 112 \Rightarrow y_1^2 = \frac{112}{37}$

and $x_1^2 = 3y_1^2 = \frac{336}{37}$

$|z_1|^2 = x_1^2 + y_1^2 = \frac{336}{37}+\frac{112}{37}=\frac{448}{37}=12.01$

$\therefore \left[|z_1|^2\right]=12$

04. (C) Let $y=\frac{ax^2-2x+3}{2x-3x^2+a}$

$\Rightarrow (a + 3y)x^2 - (2y + 2)x + (3 - ay) = 0$

$x \in R \Rightarrow (2y + 2)^2 - 4(a + 3y)(3 - ay) \geq 0$

$\Rightarrow 4y^2 + 8y + 4 - 4(3a + 9y - a^2y - 3ay^2) \geq 0$

$\Rightarrow (12a + 4)y^2 + (4a^2 - 28)y - 12a + 4 \geq 0$

$\Rightarrow (3a + 1)y^2 + (a^2 - 7)y - 3a + 1 \geq 0$

y real, $(3a + 1)y^2 + (a^2 - 7)y - 3a + 1 \geq 0$

$\Rightarrow$ The roots are real and equal or imaginary

$\Rightarrow$ Discriminant ≤ 0

$\Rightarrow (a^2 - 7)^2 + 4(3a + 1)(3a - 1) \leq 0$

$\Rightarrow a^4 - 14a^2 + 49 + 4(9a^2 - 1) \leq 0$

$\Rightarrow a^4 + 22a^2 + 45 \leq 0$

$\Rightarrow (a^2 + 11)^2 \leq 76$

$\Rightarrow a^2 + 11 \leq \sqrt{76}$

$a^2 \leq \sqrt{76}-11<0$

$\therefore$ There is no real value for a

05. (B) Number of integers greater than 6000 may be 4 digit or 5 digit

3	$^4P_3$		
6,7,8			

The number of required 4 digit numbers is $3 \times {}^4P_3 = 3 \times 24 = 72$

The number of required 5 digit numbers is ${}^5P_5 = 5! = 120$

$\therefore$ The required number of integers $= 72 + 120 = 192$

06. (A) The number of persons in a selection may be of the following types

(i) Man's relative (0 males + 3 Females) + wife's relatives (3 Males + 0 Females)

(ii) Man's relatives (1 male + 2 Females) + wife's relatives (2 males + 1 Female)

(iii) Man's relatives (2 males + 1 Female) + wife's relatives (1 male + 2 Females)

(iv) Man's relatives (3 males + 0 Females) + wife's relatives (0 males + 3 Females)

$\therefore$ The required number of selections

$$= {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0 \times {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 \times {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 \times {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3$$

$= 1 \times 4 \times 4 \times 1 + 3 \times 6 \times 6 \times 3 + 3 \times 4 \times 4 \times 3 + 1 \times 1 \times 1 \times 1$

$= 16 + 324 + 144 + 1 = 485$

07. (B) n^{th} term,

$$T_n = \frac{1+2+....+n}{1^3+2^3+....+n^3} = \frac{n(n+1)}{2} \times \frac{4}{n^2(n+1)^2}$$

$$= \frac{2}{n(n+1)} = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$S_n = \sum T_n = 2\sum\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$= 2\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + ... + \frac{1}{n} - \frac{1}{n+1}\right]$$

$$= 2\left[1 - \frac{1}{n+1}\right] = \frac{2n}{n+1}$$

$$100\,S_n = n \Rightarrow 100\left(\frac{2n}{n+1}\right) = n$$

$\Rightarrow 200 = n + 1 \Rightarrow n = 199$

08. (A) $1\,1\,1\,1 \ldots 1$ (91 times) $= 1 + 10 + 10^2 + \ldots + 10^{90}$

$$= \frac{10^{91}-1}{10-1} = \frac{\left(10^7\right)^{13}-1}{9}$$

$$= \frac{t^{13}-1}{9} = \frac{t-1}{9}\left(t^{12}+t^{11}+....+t+1\right)$$

$$= \frac{10^7-1}{10-1}\left(1+t+t^2+....+t^{12}\right)$$

$= (1 + 10 + 10^2 + ... + 10^6)(1 + t + t^2 + ... + t^{12})$ where $t = 10^7$

$\therefore$ $1\,1\,1 \ldots 1$ (91 times) is not a prime

09. (D) $$\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + ... = \sum_{n=1}^{\infty} \frac{1}{n(2n+1)}$$

$$= \sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{2}{2n+1}\right] = \sum_{n=1}^{\infty}\left[\frac{2}{2n} - \frac{2}{2n+1}\right]$$

$$= 2\sum_{n=1}^{\infty}\left[\frac{1}{2n} - \frac{1}{2n+1}\right]$$

$$= 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + ...\right]$$

$= 2[-\log_e 2 + 1] = 2 - 2\log_e 2$

10. (B) $$\underset{x\to 0}{\text{Lt}} \frac{x + 2\sin x}{\sqrt{x^2+2\sin x+1} - \sqrt{\sin^2 x - x + 1}}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{(x+2\sin x)\left[\sqrt{x^2+2\sin x+1} + \sqrt{\sin^2 x - x + 1}\right]}{(x^2+2\sin x+1) - (\sin^2 x - x + 1)}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{(x+2\sin x)}{x^2+2\sin x - \sin^2 x + x} \; \underset{x\to 0}{\text{Lt}}\left[\sqrt{x^2+2\sin x+1} + \sqrt{\sin^2 x - x + 1}\right]$$

$$= \underset{x\to 0}{\text{Lt}} \frac{1+2\cos x}{2x+2\cos x - \sin 2x + 1} \times 2$$

$$= \frac{3}{3} \times 2 = 2$$

11. (C) $y = \sec(\text{Tan}^{-1} x)$

$= \sqrt{\sec^2 \text{Tan}^{-1} x}$

$= \sqrt{1+\text{Tan}^2 \text{Tan}^{-1} x}$

$= \sqrt{1+x^2}$

$\frac{dy}{dx} = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{2}}$

12. (B)

class interval	Midpoints(x_i)	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0-10	5	4	20	30	120
10-20	15	6	90	20	120
20-30	25	16	400	10	160
30-40	35	28	980	0	0
40-50	45	16	720	10	160
50-60	55	6	330	20	120
60-70	65	4	260	30	120
		80	2800		800

Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2800}{80} = 35$

Mean deviation about mean

$s = \frac{\sum f_i \lvert x_i - \bar{x}\rvert}{\sum f_i} = \frac{800}{80} = 10$

13. (C) $n(S) = {}^6C_3 = 20$, n(A) = 2

[In the hexagon ABCDEF; ACE, BDF are equilateral]

$\Rightarrow P(A) = \frac{2}{20} = \frac{1}{10}$

14. (B) Let a = tan α, b = tan β, c = tan γ

Then $a^2b^2 + b^2c^2 + c^2a^2 + 2a^2b^2c^2 = 1$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2a^2b^2c^2 + 1 + a^2 + b^2 + c^2 - a^2b^2c^2$

$= 1 + 1 + a^2 + b^2 + c^2 - a^2b^2c^2$

$1 + a^2 + b^2 + c^2 + a^2b^2 + b^2c^2 + c^2a^2 + a^2b^2c^2 = 2 + a^2 + b^2 + c^2 - a^2b^2c^2$

$\Rightarrow (1 + a^2)(1 + b^2)(1 + c^2) = 2(a^2b^2 + b^2c^2 + c^2a^2 + 2a^2b^2c^2) + a^2 + b^2 + c^2 - a^2b^2c^2$

$= a^2 + b^2 + c^2 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2 + 3a^2b^2c^2$

$= a^2(1 + b^2)(1 + c^2) + b^2(1 + c^2)(1 + a^2) + c^2(1 + a^2)(1 + b^2)$

$\Rightarrow \frac{a^2}{1+a^2} + \frac{b^2}{1+b^2} + \frac{c^2}{1+c^2} = 1$

$\Rightarrow \frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma} = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

15. (B)

(cot B + cot C)(cot C + cot A)(cot A + cot B)

$= \left(\frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right)\left(\frac{\cos C}{\sin C} + \frac{\cos A}{\sin A}\right)\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}\right)$

$= \frac{\sin(B+C)}{\sin B \sin C} \times \frac{\sin(C+A)}{\sin C \sin A} \times \frac{\sin(A+B)}{\sin A \sin B}$

$= \frac{\sin(180° - A)}{\sin B \sin C} \times \frac{\sin(180° - B)}{\sin C \sin A} \times \frac{\sin(180° - C)}{\sin A \sin B}$

$= \frac{\sin A \sin B \sin C}{\sin B \sin C \times \sin C \sin A \times \sin A \sin B}$

$= \frac{1}{\sin A \sin B \sin C}$

16. (A)

$x = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{7\pi}{15}\cos\frac{30\pi}{15}$

$= \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\frac{1}{2}\cos\left(\pi - \frac{8\pi}{15}\right)1$

$= \frac{-1}{4\sin\left(\frac{\pi}{15}\right)}\left(2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\right)\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

$= -\frac{1}{4\sin\left(\frac{\pi}{15}\right)}\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

$$= -\frac{1}{8\sin(\pi/15)}\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$$

$$= -\frac{1}{16\sin(\pi/15)}\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}$$

$$-\frac{1}{32\sin(\pi/15)}\sin\frac{16\pi}{15} = \frac{\sin(\pi/15)}{32\sin(\pi/15)}$$

$$= \frac{1}{32} \Rightarrow \frac{1}{8x} = 4$$

17. (C) $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$

$$= \frac{1}{2\sin\left(\frac{2\pi}{15}\right)}\left(2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\right)\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$$

$$= \frac{1}{4\sin\left(\frac{2\pi}{15}\right)}\left(2\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\right)\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$$

$$= \frac{1}{8\sin\left(\frac{2\pi}{15}\right)}\left(2\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\right)\cos\frac{14\pi}{15}$$

$$= \frac{1}{8\sin\left(\frac{2\pi}{15}\right)}\sin\frac{16\pi}{15}\cos\frac{14\pi}{15}$$

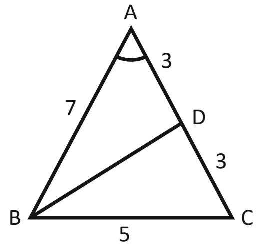
$$= \frac{1}{16\sin\left(\frac{2\pi}{15}\right)}2\sin\left(\pi+\frac{\pi}{15}\right)\cos\left(\pi-\frac{\pi}{15}\right)$$

$$= \frac{1}{16\sin\left(\frac{2\pi}{15}\right)}\left(2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\right) = \frac{\sin\frac{2\pi}{15}}{16\sin\frac{2\pi}{15}} = \frac{1}{16}$$

18. (A)

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{36+49-25}{2(6)(7)} = \frac{60}{84} = \frac{5}{7}$$

Let BD be the median through B



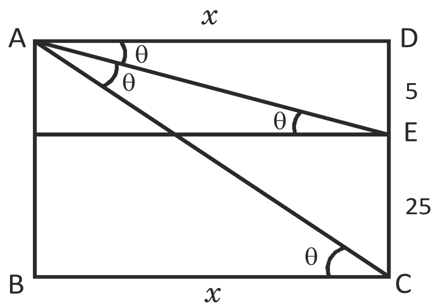
$BD^2 = AD^2 - 2(AB)(AD)\cos A$

$= 49 + 9 - 2 \times 7 \times 3 \times \frac{5}{7} = 58 - 30 = 28$

$\therefore$ BD = $\sqrt{28} = 2\sqrt{7}$

19. (D) In ΔADE $\tan\theta = \frac{5}{x}$, and in ΔACD

$\tan 2\theta = \frac{30}{x} = 6 \times \frac{5}{x}$



$$\tan 2\theta = 6\tan\theta$$

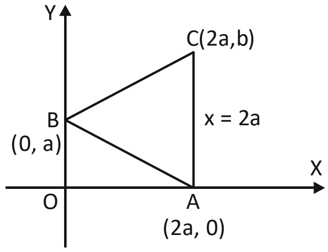
$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = 6\tan\theta \Rightarrow 3 - 3\tan^2\theta = 1$$

$$\Rightarrow \tan^2\theta = \frac{2}{3} \Rightarrow \tan\theta = \sqrt{\left[\frac{2}{3}\right]}$$

$$\Rightarrow \frac{5}{x} = \sqrt{\frac{2}{3}} \Rightarrow x = \frac{5\sqrt{3}}{\sqrt{2}}$$

20. (B) Let y-coordinate of C = b

∴ C = (2a, b)



$AB = \sqrt{4a^2 + a^2} = \sqrt{5}a$

Now, $AC = BC \Rightarrow b = \sqrt{4a^2 + (b-a)^2}$

$\Rightarrow b^2 = 4a^2 + b^2 + a^2 - 2ab$

$\Rightarrow 2ab = 5a^2 \Rightarrow b = \frac{5a}{2}$

$\therefore C = \left(2a, \frac{5a}{2}\right)$

A(2a, 0) B(0, a) & C(2a, $\frac{5a}{2}$)

Area of $\Delta ABC = \frac{1}{2}\left|2a\left(a - \frac{5a}{2}\right) + 0\left(\frac{5a}{2}\right)\right|$

$= \frac{1}{2}\left|-3a^2 - 2a^2 = \frac{5a^2}{2} + 2a(0-a)\right|$

$= \frac{1}{2} \times 2a\left(-\frac{5a}{2}\right) = -\frac{5a^2}{2}$

Since area is always +ve, hence area

$= \frac{5a^2}{2}$ sq. unit

21. (C) Truth table of all options is as follows

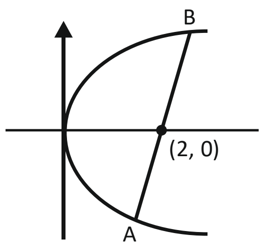
p	q	$p \vee q$	$p \wedge q$	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$	$p \to q$
T	F	T	F	T	T	F
F	T	T	F	F	F	T
T	T	T	T	T	T	T
F	F	F	F	F	T	T

$p \wedge (p \to q)$	$[(p \wedge (p \to q) \to q]$	$[q \to [p \wedge (p \to q)]]$
F	T	T
F	T	F
T	T	T
T	T	T

∴ It is tatology.

22. (B) Let parabola $y^2 = 8x$ at point $\left(\frac{1}{2}, -2\right)$ is $(2t^2, 4t)$

$\Rightarrow t = \frac{-1}{2}$



$\left(\frac{1}{2}, -2\right)$

Parameter of other end of focal chord is 2

So, coordinates of B is (8, 8)

⇒ Equation of tangent at B is

$8y - 4(x + 8) = 0$

$\Rightarrow 2y - x = 8$

$\Rightarrow x - 2y + 8 = 0$

23. (D) Given inequally is,

$$2\sqrt{\sin^2 x-2\sin x+5}\le 2\times 2\sin^2 y$$

$$\Rightarrow \sqrt{\sin^2 x-2\sin x+5}\le 2\sin^2 y$$

$$\Rightarrow \sqrt{(\sin x-1)^2+4}\le 2\sin^2 y$$

It is true if sin x = 1 and |sin y| = 1

Therefore, sin x = |sin y|

24. (C) Given, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=p$ (let) and point P(β,0,β)

Any point on line A = (p, 1, −p − 1)

Now, DR of APa''

< p − β, 1 − 0, −p −1 − β >

Which is perpendicular to line.

∴ (p − β)1 + 0.1 − 1(−p − 1 −β) = 0

$$\Rightarrow p=\beta+p+1+\beta=0\Rightarrow p=\frac{-1}{2}$$

$$\therefore \text{ Point } A\left(\frac{-1}{2},1-\frac{1}{2}\right)$$

Given that distance AP

$$=\sqrt{\frac{3}{2}}\Rightarrow AP^2=\frac{3}{2}$$

$$\Rightarrow\left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2=\frac{3}{2}$$

$$\text{or }\ 2\left(\beta+\frac{1}{2}\right)^3=\frac{1}{2}$$

$$\Rightarrow\left(\beta+\frac{1}{2}\right)^2=\frac{1}{4}\Rightarrow\beta=0,-1,(\beta\ne 0)$$

$$\therefore\ \beta=-1$$

25. (C) Given

$$\log_{(2^2)}(2^{1-x}+1)=\frac{\log_2(5\times 2^x+1)+\log_2 2}{2}$$

$$\frac{1}{2}\log_2(2^{1-x}+1)=\frac{1}{2}\left[\log_2(5\times 2^x+5)2\right]$$

$$2^{1-x}+1=10\times 2^x+2$$

$$\Rightarrow\quad \frac{2}{2^x}+1=10\times 2^x+2$$

let 2^x = a ⇒ $\frac{2}{a}$ = 10a + 2 − 1

$\frac{2}{a}$ = 10a + 1

2 = 10a² + a

10a² + a − 2 = 0

10a² + 5a − 4a − 2 = 0

5a(2a + 1) − 2(2a + 1) = 0

∴ 5a − 2 = 0

5a = 2

a = $\frac{2}{5}$

2^x = $\frac{2}{5}$

$$\Rightarrow\quad \log_2\left(\frac{2}{5}\right)=x$$

$$\text{i.e. } \frac{\log 2-\log 5}{\log 2}=x$$

PHYSICS

26. (C) Distance travelled in n second is equal to $n\nu_0 + \frac{1}{2}an^2$

Distance travelled in (n – 1) second is equal to (n – 1) $\nu_0 + \frac{1}{2}a(n-1)^2$

Distance travelled in n^{th} second is

$$x_n = \left[n\nu_0 + \frac{1}{2}an^2\right] - \left[(n-1)\nu_0 + \frac{1}{2}a(n-1)^2\right]$$

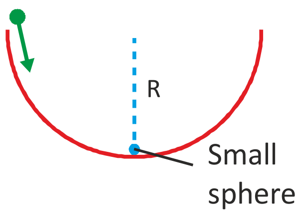
$$x_n = \nu_0 + \frac{1}{2}a(2n-1)$$

27. (Delete)

28. (D) Given, stress in steel wire = Stress in brass wire so,

$$\frac{T_1}{A_1} = \frac{T_2}{A_2} \text{ or } \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2}$$

29. (D) It is clear from the figure given below.



On reaching the bottom of the bowl, loss in P.E. = mg R, and

Gain in K.E. = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\left(\frac{2}{5}mr^2\right)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

As gain in K.E. = Loss in P.E.

$$\therefore \quad \frac{7}{10}mv^2 = mgR$$

$$v = \sqrt{\frac{10\,gR}{7}}$$

30. (B) $u = \frac{3GM^2}{5R}$

$$u' = \frac{3}{5}\times\frac{GM^2}{R} = \frac{3GM^2}{5R/2}$$

$$= \frac{3\times 2}{5}\times\frac{GM^2}{R}$$

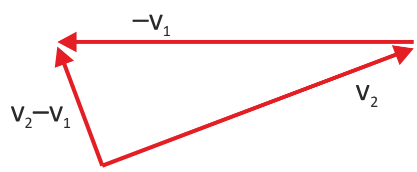
$$u' - u = \frac{3}{5}\frac{GM^2}{R}$$

$$= \frac{3}{5}\times\frac{6.67\times10^{-11}\times(2\times10^{30})^2}{7\times10^8}$$

$$= \frac{(3\times6.67\times4\times10^{60})}{5\times7\times10^8}$$

$= 2.3 \times 10^{41}$ J

31. (C) By definition, $\bar{a} = \frac{\Delta v}{\Delta t}$. We determine $\Delta v = v_2 - v_1 = v_2 + (-v^1)$ geometrically as follows:



As Δt is a positive scalar, the direction of $\bar{a}$ is the same as the direction of Δv, which is displayed above.

32. (D) Gravitational potential energy,

$$E_P = \frac{GMm}{(R_e + R_e)} = -\frac{GMm}{2R_e} = -\frac{1}{2}mg\,R_e$$

33. (C) Pressure inside the cavity

$= P_0 + h\rho g + \frac{2S}{r}$

$= 0.76 \times (13.6 \times 10^3) \times (9.8) + 0.2 \times 0.85 \times 10^3 \times 9.8 + \frac{2\times26\times10^{-3}}{13\times10^{-6}}$

$= 1.013 \times 10^5 + 0.0167 \times 10^5 + 0.04 \times 10^5$

$= 1.0697 \times 10^5$ N m^{-2} or 1.07×10^5 N m^{-2}.

34. (C) In a time t_0, the displacement of the block with respect to ground is $\frac{1}{2}at_0^2$ (downward). Therefore, the work done by mg is W = $mg\left(\frac{1}{2}at_0^2\right) = \frac{1}{2}mg\,at_0^2$

35. (A) The sphere has the least surface area, therefore, it takes the longest time to cool down.

The circular plate with the maximum surface area is the fastest to cool down.

36. (C) Let s be the total length of trains and u_1, u_2 be their speeds. As per question,

$$\frac{s}{u_1+u_2} = 3 \text{ and } \frac{s}{\frac{3}{2}u_1+u_2} = \frac{5}{2}$$

or $3u_1 + 3u_2 = \frac{5}{2}\times\frac{3}{2}u_1 + \frac{5}{2}u_2$

or $12u_1 + 12u_2 = 15u_1 + 10u_2$

or $3u_1 = 2u_2$ or $u_1 = \frac{2}{3}u_2$

If t is the time taken to cross distance s when the train pass in the same direction, then

$$t = \frac{s}{u_2-u_1} = \frac{3(u_1+u_2)}{u_2-u_1}$$

$$= \frac{3\left(\frac{2}{3}u_2+u_2\right)}{u_2-\frac{2}{3}u_2} = \frac{3\times5}{3-2} = 15\,s$$

37. (D) Internal energy U = No. of moles × No. of degrees of freedom = $\frac{1}{2}RT$

Out of four cases, product of no. of moles (1000) degrees of freedom (3) and T (= 900 K) is maximum for argon gas.

38. (C) m_1 = 1 kg, $m_2 = 6\times10^{24}$ kg

Force of attraction = F = ?

G = 6.67×10^{-11} Nm^2 kg^{-2}

Distance between the two masses = r = 6.38×10^6 m

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times6\times10^{24}}{(6.38\times10^6)^2} = 9.83\text{ N}$$

Acceleration of $m_1 = a_1 = \frac{\text{Force}}{m_1}$

$$a_1 = \frac{9.83}{1} = 9.83\text{ m/s}^2$$

Acceleration of $m_2 = a_2 = \frac{\text{Force}}{m_2} = \frac{9.83}{6\times10^{24}}$

$a_2 = 1.64\times10^{-24}$ m/s^2

39. (A) $\frac{mg}{\eta r} = \frac{M(LT^{-2})}{(ML^{-1}T^{-1})L} = LT^{-1} = v_T$

40. (A) Here, h_1 = 550 m, h_2 = 50 m

m = 2000 kg, t = 1 sec

Efficiency = 80%

∴ Maximum electrical power output

$$= \frac{80}{100}\times\frac{mg(h_1-h_2)}{t}$$

$$= \frac{4}{5}\times\frac{2000\times10(550-50)}{1}$$

P = 8×10^6 watt = 8 MW.

CHEMISTRY

41. (A) 1 E + EA = 275 + 86 = 361 k cal mol^{-1}

= 361 × 4.184

= 1510.42 kJ mol^{-1}

∴ Electronegativity

$= \frac{1510.42}{540} = 2.797 = 2.8$

42. (D) $\underset{1-\alpha}{N_2O_4} \rightleftharpoons \underset{2\alpha}{2NO_2}$ Total = 1 + α

43. (C) There is no restriction that resonating structures should have +ve and –ve charges on atoms that are far apart.

44. (D) $\overset{6}{C}H_3\overset{5}{C}H_2 - \overset{4}{C}H = \overset{3}{C}H - \overset{2}{C}(=O) - \overset{1}{C}H_3$

(3–Hexen–5–one)

45. (C) Mass of NaOH required = 0.184 g

Molar mass of NaOH = 40 g/mol

Molarity of NaOH solution = 0.150 mol^{-1}

Let, V be the volume of NaOH required to be added into the reaction vessel. Then

Amount of NaOH added = $M_{NaOH} \times V_{NaOH}$

From the given data,

$\frac{0.184\ g}{40\ g/mol} = 0.150\ mol\ L^{-1} \times V$

$V = \frac{0.184}{40} mol \times \frac{1}{0.150\ mol\ L^{-1}} = \frac{0.184}{40 \times 0.150} L$

$= \frac{0.184 \times 1000}{40 \times 0.150} mL = 30.7\ mL$

46. (B) Half-cell reaction :

Cu^{2+} (aq) + $2e^-$ ⟶ Cu

The half-cell potential is given by,

$E = E° + \frac{0.059\ V}{2} \log [Cu^{2+}]$

E = E° + (0.0295 V) log $[Cu^{2+}]$

The half-cell potential when the solution is diluted 100 times

E = E° + (0.0295 V) log $(10^{-2} [Cu^{2+}])$

Thus, E' – E = (0.0295 V) log 10^{-2} = – 0.059 V

The potential of half cell will decrease by 59 mV.

47. (A) We have $\Delta x \cdot (m\,\Delta v) = \frac{h}{4\pi}$

or $m = \frac{h}{4\pi} \times \frac{1}{\Delta x \cdot \Delta v}$

$= \frac{6.625 \times 10^{-34} (kg \cdot m^2\ s^{-1})}{4 \times 3.14 \times (10^{-10} m)(5.27 \times 10^{-24}\ m\,s^{-1})}$

= 0.10 kg

48. (D) Due to thunderstorm, temperature increases, i.e., $[H^+]$ increases which means pH decreases.

49. (A) $C : H : Cl : O = \frac{18.5}{12} : \frac{1.55}{1} : \frac{55.04}{35.5} : \frac{24.81}{16}$

= 1.54 : 1.55 : 1.55 : 1.55

∴ E.F. = CHC*l*O.

50. (B) Rise in temperature,

Δt = (300.78 K – 294.05 K) = 6.73 K

Heat capacity of the calorimeter

= 8.93 kJ K^{-1}

Then,

Heat transferred to calorimeter = Heat capacity of calorimeter × Rise in temperature = 8.93 kJ K^{-1} × 6.73 K = 60.1 kJ

51. (B) 6 g $O_2 = \frac{6}{32}$ mol = 0.1875,

6 g $SO_2 = \frac{6}{64}$ mol = 0.09375

As no. of moles of SO_2 is less, so the no. of molecules will also be less.

52. (A) μ (100% ionic)

= (1.602×10^{-19} C) × (1.6×10^{-10} m)

= 2.56×10^{-29} C m

∴ % of Ionic character $= \frac{2.0 \times 10^{-29}}{2.56 \times 10^{-29}} \times 100$ = 78%

53. (A) $r = \dfrac{n^2 h^2}{4\pi^2 m Ze^2}$. Here m = mass of e^-.

Mass of atom is not involved.

As Z = 1 for both, ratio = 1 : 1

54. (C) For the reaction,

$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$

$K_p = 1.80 \times 10^{-3}$ kPa

$K_c = ?$

We know that,

$K_p = K_c(RT)^{\Delta n}$

Where $\Delta n = \Sigma n_g \text{ (products)} - \Sigma n_g \text{ (reactants)}$

For the given reaction,

$\Delta n = 2 + 1 - 2 = 1$

So, $K_p = K_c \times RT$

R = 0.082 L atm K^{-1} mol^{-1} = 0.082 L atm

$K^{-1}\ mol^{-1} \times \dfrac{101.3\text{ kPa}}{1\text{ atm}} = 8.31\text{ L kPa } K^{-1}\ mol^{-1}$

Then,

$K_c = \dfrac{K_p}{RT} = \dfrac{1.80 \times 10^{-3}\text{ kPa}}{8.31\text{ L kPa } K^{-1}\ mol^{-1} \times 700\text{ K}}$

$= 3.09 \times 10^{-7}$ mol L^{-1}

55. (C) The outer electronic configuration of group 15 elements is s^2p^3.

CRITICAL THINKING

56. (A) Starting from the outer end of the spiral (the loop on the rope) the green and white sections are longest, and the white sections are of similar length to the green sections. As you move towards the other end of the rope, both green and white sections get shorter. Only rope (A) shows this, hence (A).

57. (C) We can see from the image that the tube is coming out of the upper wall of the first pool; this means that the second pool will only start to fill up after the first pool is completely full. We need to total the amount of time it will take to fill up each pool.

The volume of pool $1 \times 1 \times 3.6[m^3] \times 1000$

= 3600 liter

The volume of pool $1 \times 1 \times 2 \times 0.6[m^3] \times 1000$ = 600 liter

The time taken to fill pool 1:

3600 liter / 1 liter/second

= 3600 seconds = 1 hour

The time taken to fill pool 2:

600 liter / 1 liter/second

= 600 seconds = 10 minutes

Therefore, the total amount of time: 60 min + 10 min = 70 minutes

58. (C) S1 and S2 together are not sufficient to answer the Question.

59. (B) S – R. S lives in hostel & R lives in house.

Students	Subject	Living place
P	English	Paying guest
Q	History	House
R	Philosophy	House
S	Physics	Hostel
T	Mathematics	Paying guest
U	Commerce	Hostel

60. (B)